

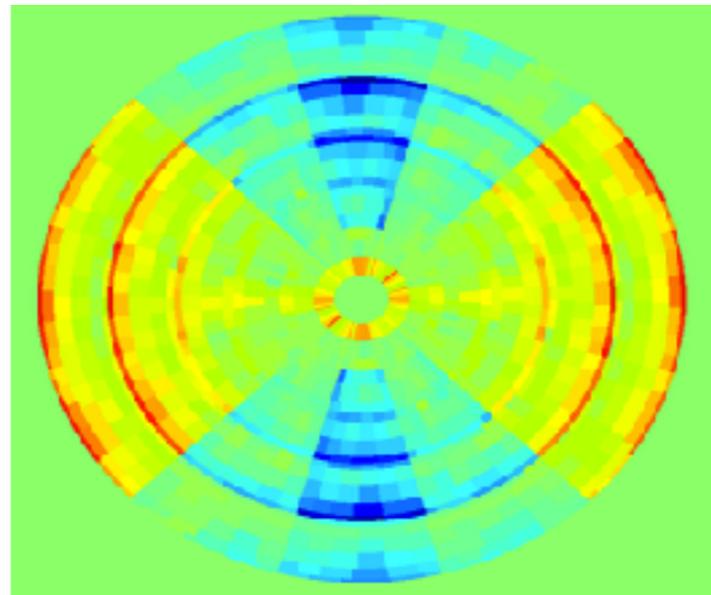
Organising Deep Networks

RDMath IdF

Domaine d'Intérêt Majeur (DIM)
en Mathématiques

 **île de France**

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advisor: Stéphane Mallat

following the works of Laurent Sifre, Joan Bruna, ...

collaborators: Eugene Belilovsky, Sergey Zagoruyko, Jörn Jacobsen, ...

High Dimensional classification

$$(x_i, y_i) \in \mathbb{R}^{224^2} \times \{1, \dots, 1000\}, i < 10^6 \longrightarrow \hat{y}(x)?$$



Estimation problem



"Rhino"

Training set to predict labels



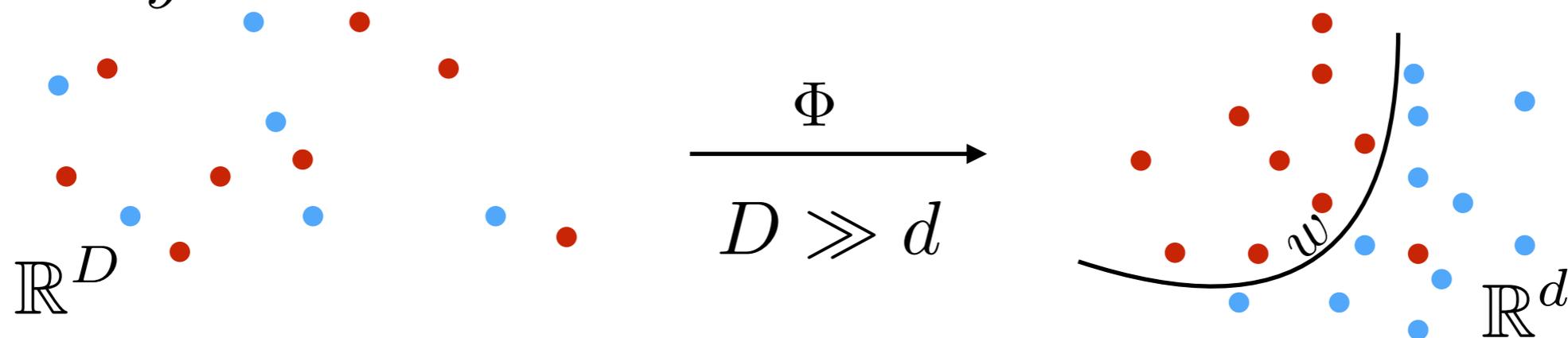
"Rhinos"



Not a "rhino"

Fighting the curse of dimensionality

- **Objective:** building a representation Φx of x such that a simple (say euclidean) classifier \hat{y} can estimate the label y :

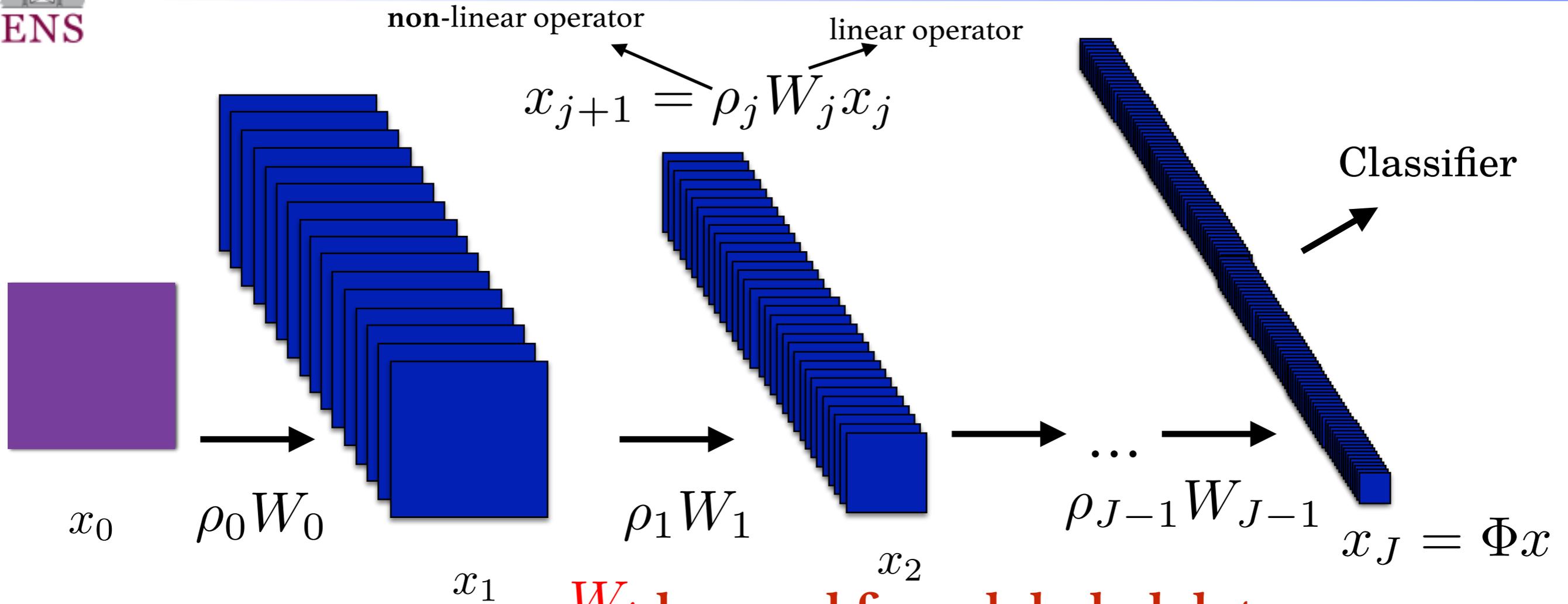


- Designing Φ consist of building an approximation of a low dimensional space which is regular with respect to the class:

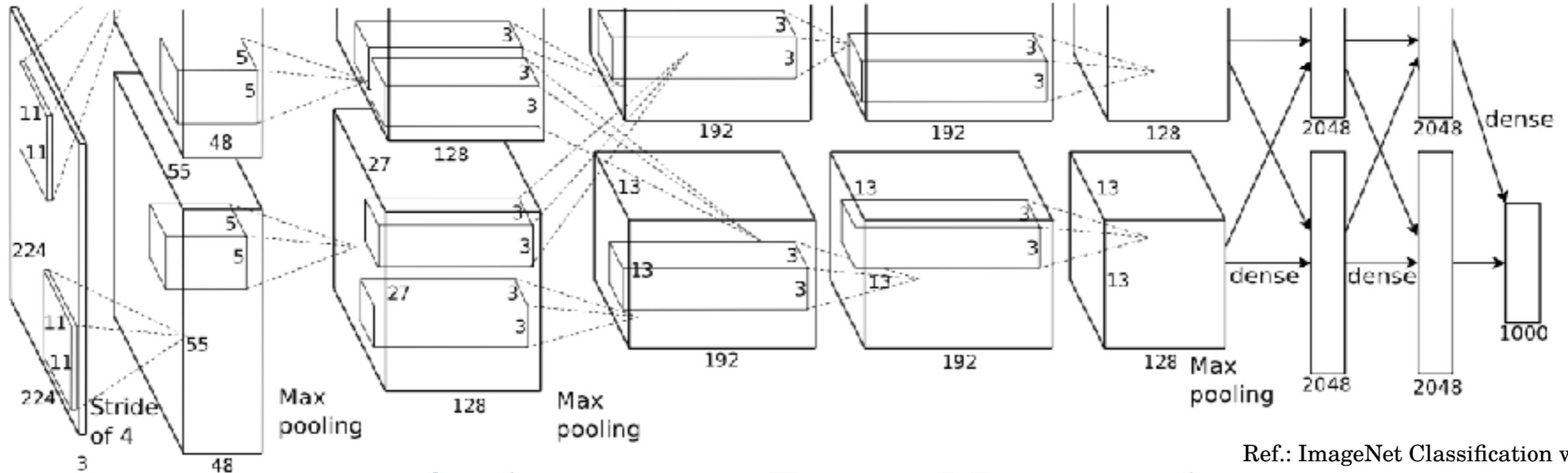
$$\|\Phi x - \Phi x'\| \lll 1 \Rightarrow \hat{y}(x) = \hat{y}(x')$$

- **Necessary dimensionality** and variance reduction

Completely solved by the deep blackbox



W_i learned from labeled data



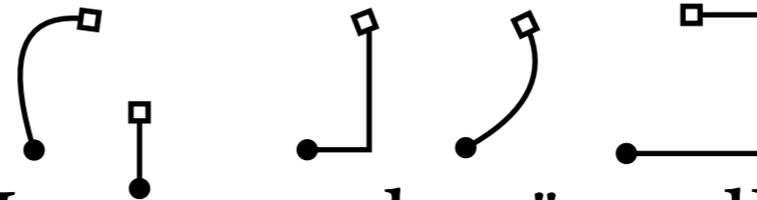
Solving it: DeepNetwork

Ref.: ImageNet Classification with Deep Convolutional Neural Networks. A Krizhevsky et al.

Why mathematics about deep learning are important

- **Pure black box.** Few mathematical results are available. Many rely on a "manifold hypothesis".

Ex: stability to diffeomorphisms



- **No stability results.** It means that "small" variations of the inputs might have a large impact on the system. And this happens.

Ref.: Intriguing properties of neural networks.
C. Szegedy et al.

- **No generalisation result.** Rademacher complexity can not explain the generalization properties.

Ref.: Understanding deep learning requires rethinking generalization
C. Zhang et al.

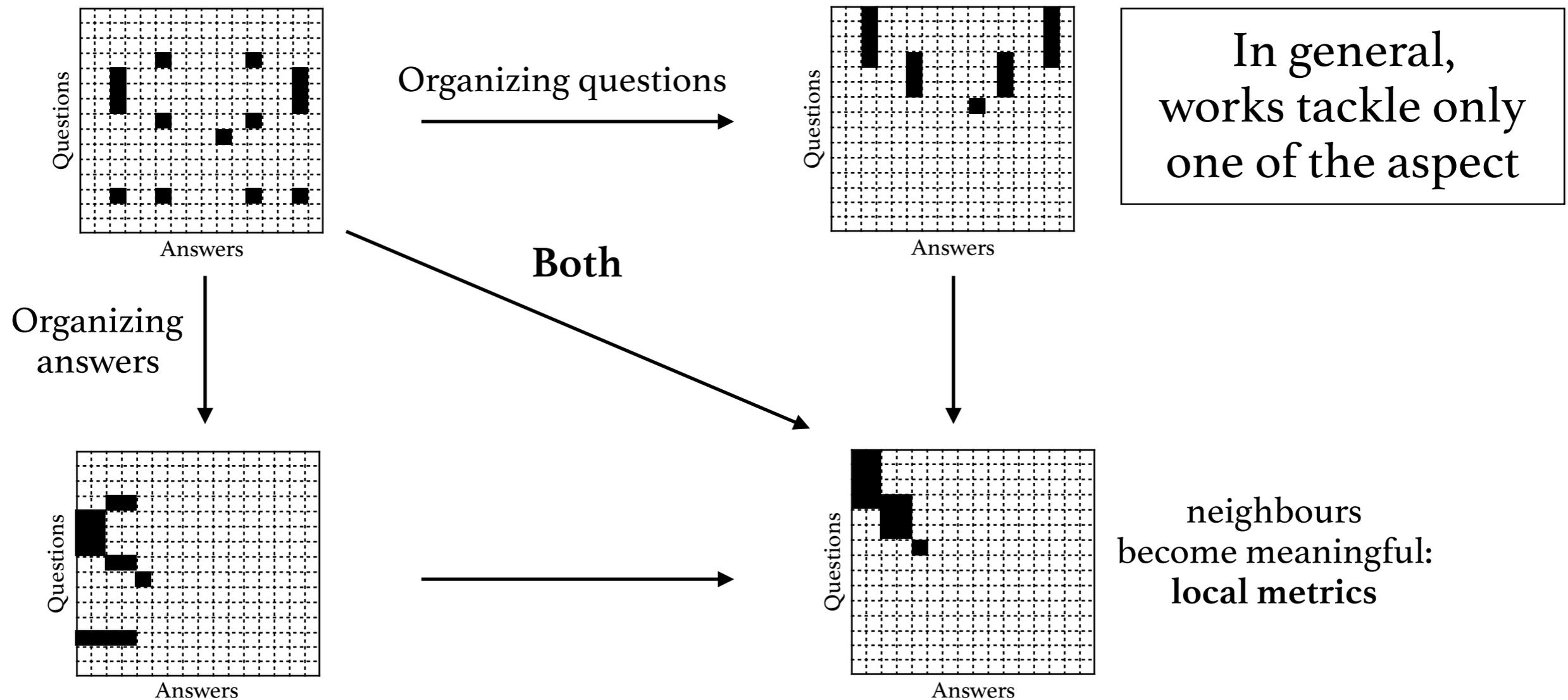
- Shall we learn each layer from scratch? (geometric priors?) The deep cascade makes features are hard to interpret

Ref.: Deep Roto-Translation Scattering for Object Classification. EO and S Mallat

Organization is a key

- Consider a problem of questionnaires: people answer to 0 or 1 to some question. What does organizing mean?

Ref.: Harmonic Analysis of Digital Data Bases
Coifman R. et al.



Structuring the input with the Scattering Transform

- Scattering Transform S_J is a deep local descriptor of neighbourhood of amplitude 2^J , for images.

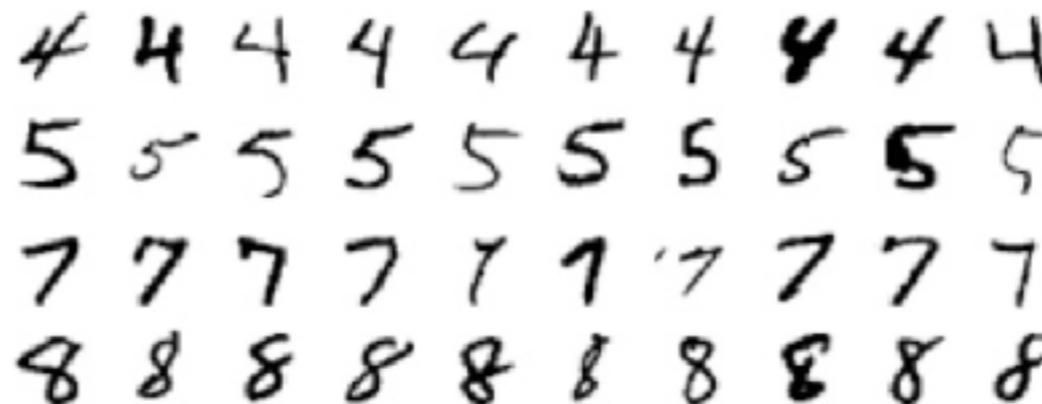
Ref.: Group Invariant Scattering, Mallat S

- It is a representation built via geometry with limited learning. (~SIFT)

Ref.: Invariant Convolutional Scattering Network, J. Bruna and S Mallat

- Successfully used in several applications:

- Digits



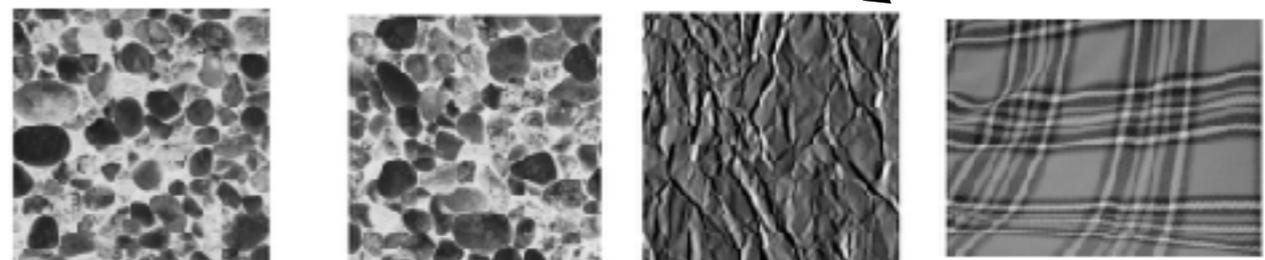
All variabilities are known

Small deformations + Translation

Rotation+Scale

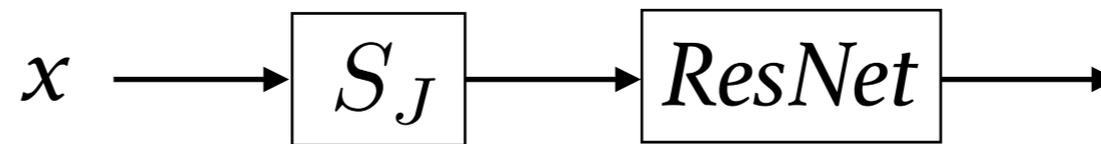
Ref.: Rotation, Scaling and Deformation Invariant Scattering for texture discrimination, Sifre L and Mallat S.

- Textures



Scattering on ImageNet: Geometry in CNNs

Ref.: Scaling the Scattering Transform:
Deep Hybrid Networks
EO, E Belilovsky, S Zagoruyko



- Cascading a modern CNN leads to almost state-of-the-art result on Imagenet2012:

1M images for training, 400k testing, 1000 classes

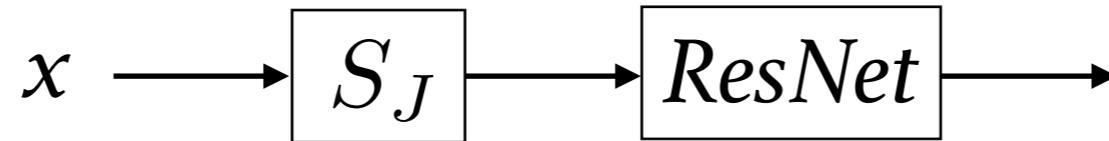
	Accuracy	Depth	#params
AlexNet	80.1	9	61M
ResNet	88.8	18	11.7M
Scat+ResNet	88.6	10	12.8M

- Demonstrates no loss of information + Less layers

Learning?

Benchmarking

Scattering + small data

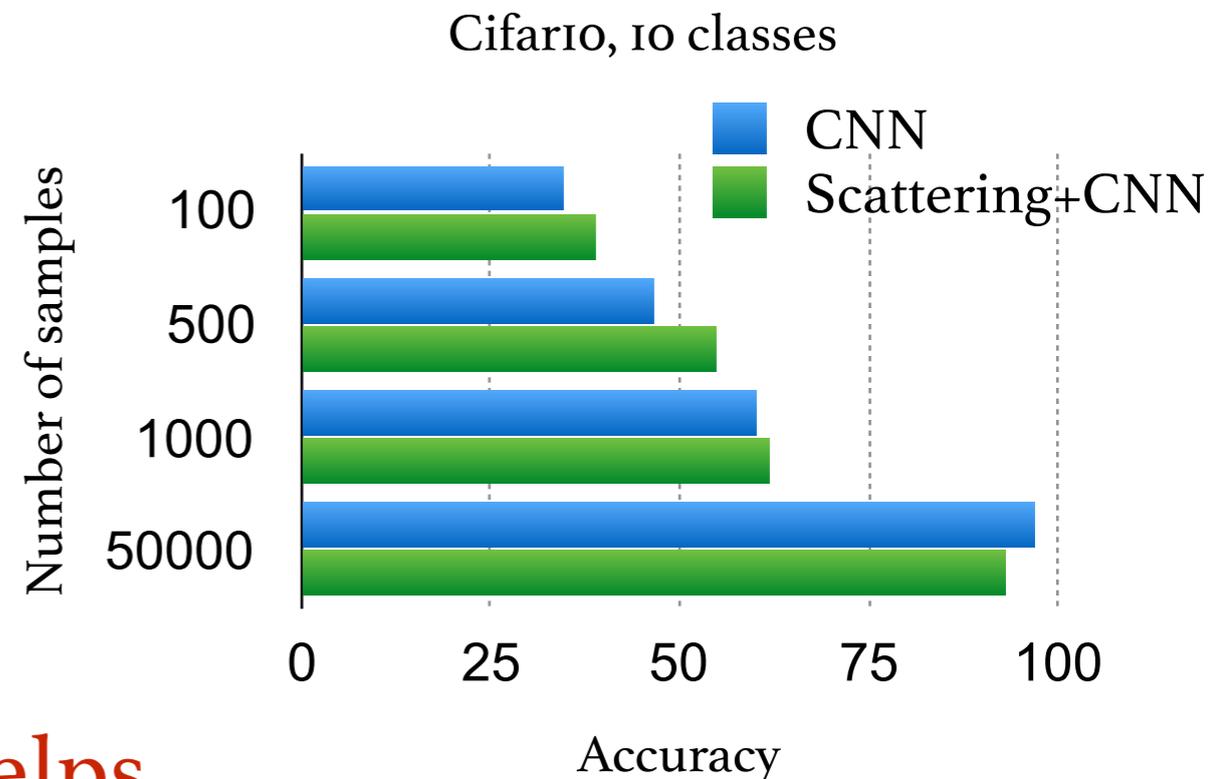


Ref.: Scaling the Scattering Transform:
Deep Hybrid Networks
EO, E Belilovsky, S Zagoruyko

- Adding geometric prior regularises the CNN input, in the particular case of limited samples situations, **without reducing the number of parameters.**
- State-of-the-art results on STL10 and CIFAR10:

STL10: 5k training, 8k testing, 10 classes
+100k unlabeled(not used!!)

	Accuracy
Scattering+CNN	76
Deep	70
Unsupervised	75



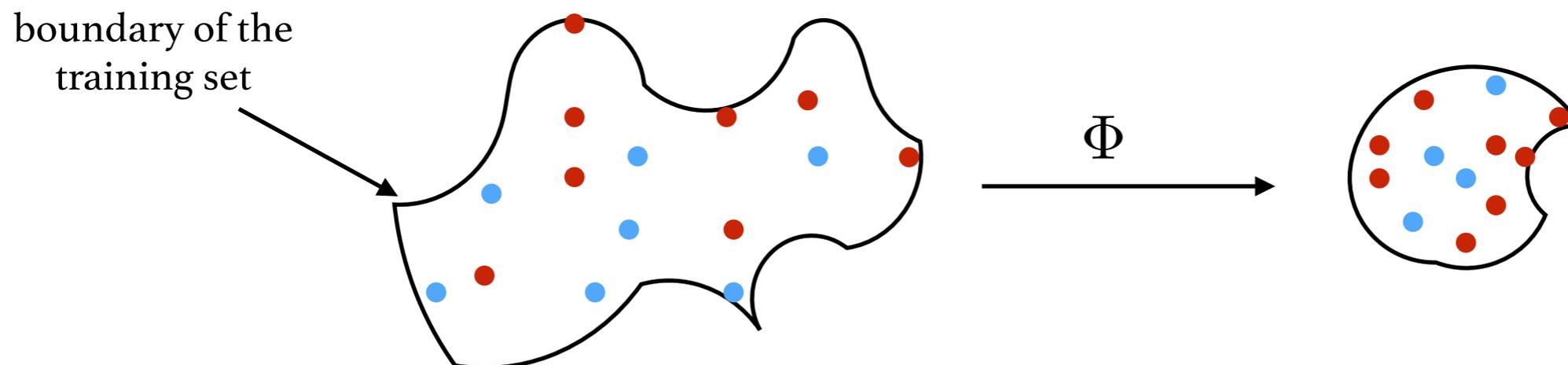
Geometry helps

Necessary mechanism: Separation - Contraction

- In high dimension, typical distances are huge, thus an appropriate representation must contract the space:

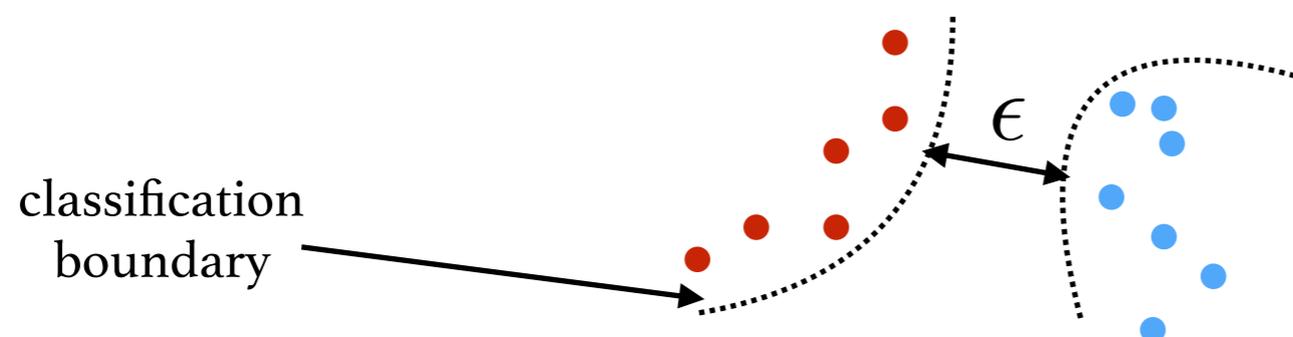
$$\|\Phi x - \Phi x'\| \leq \|x - x'\|$$

Ref.: Understanding deep convolutional networks
S Mallat



- While avoiding the different classes to collapse:

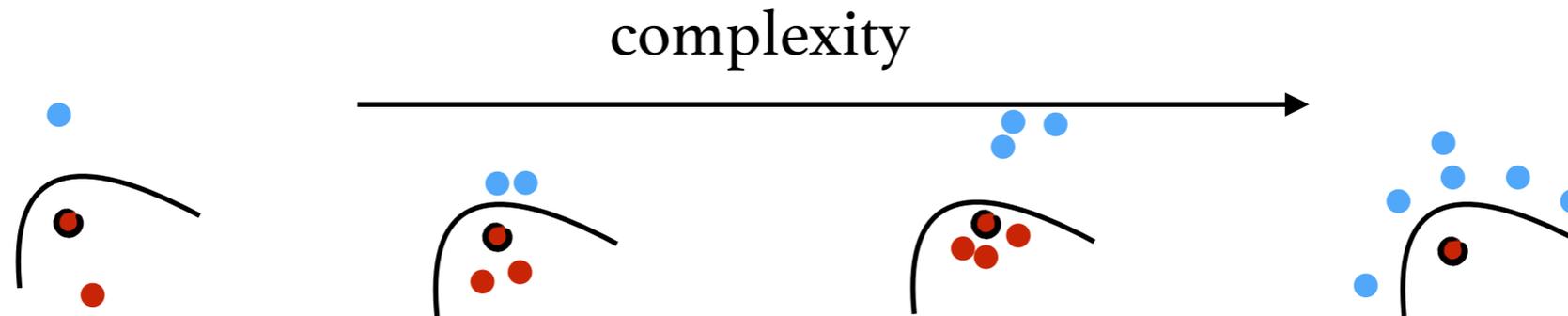
$$\exists \epsilon > 0, y(x) \neq y(x') \Rightarrow \|\Phi x - \Phi x'\| \geq \epsilon$$



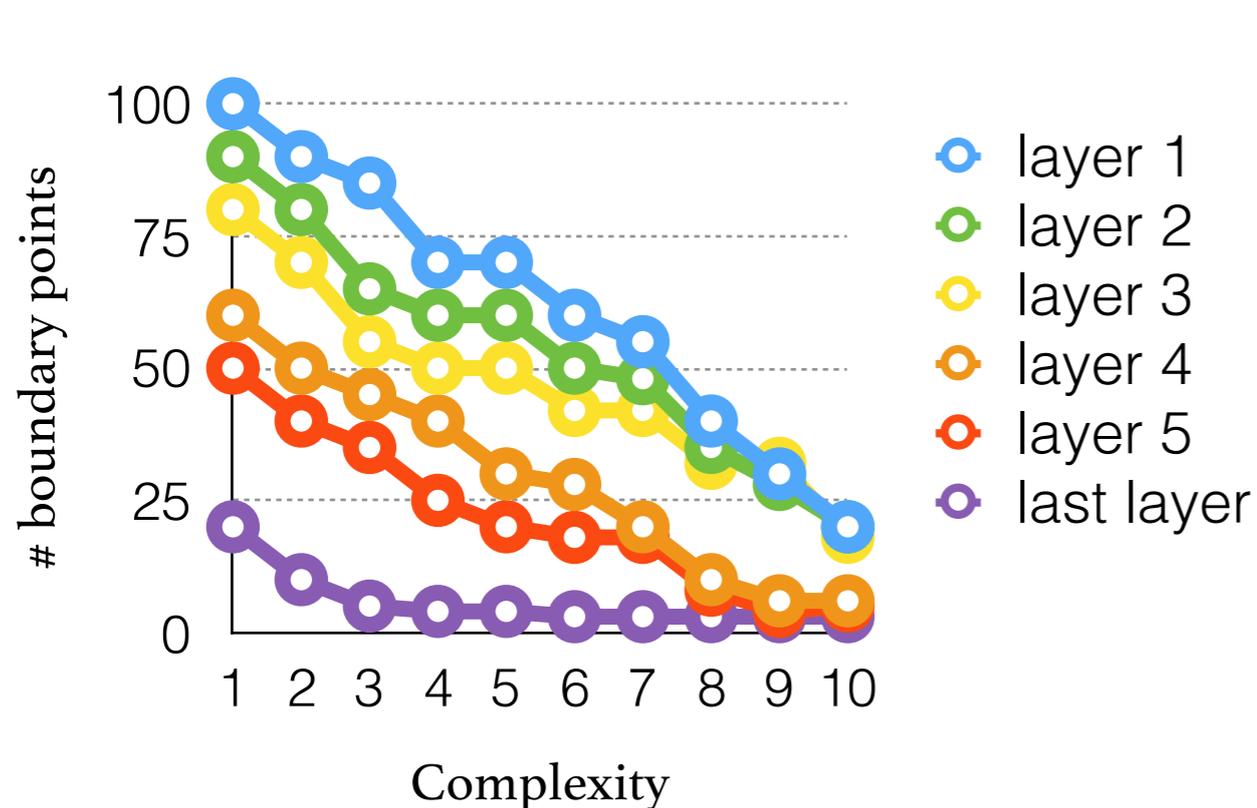
Complexity measure

- Measuring the complexity of the classification boundary (estimating the local dimensionality is hard)

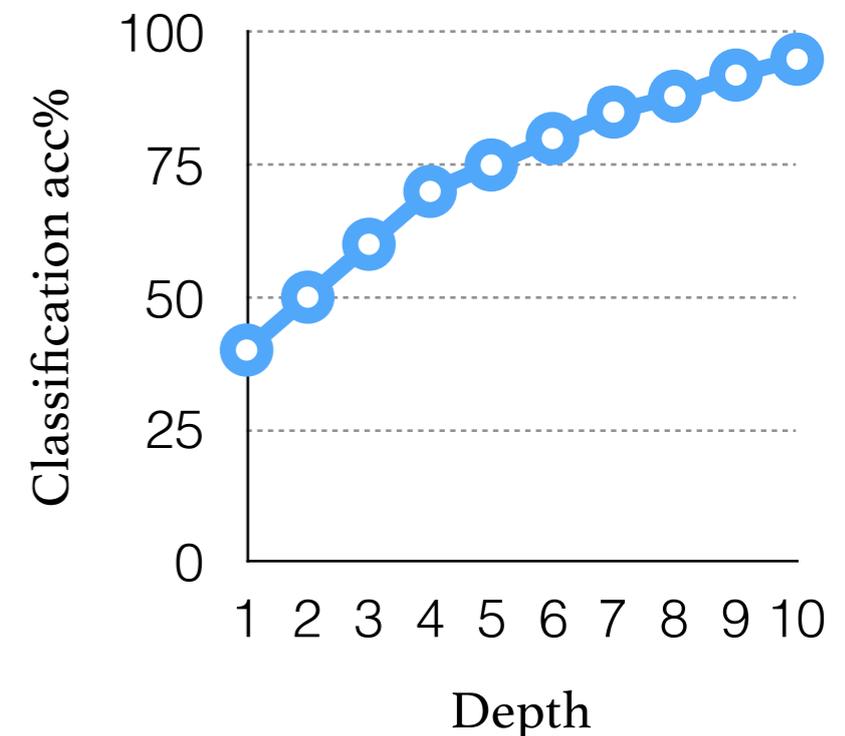
Ref.: Building a Regular Decision Boundary with Deep Networks
EO



- Progressive contraction of the space, at each layer:



Explains the improvement

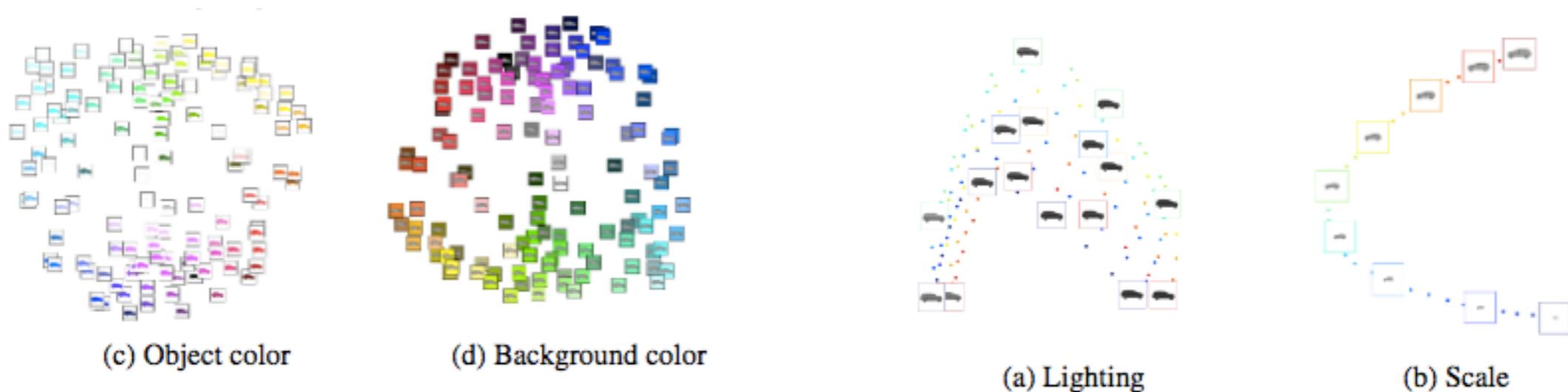


What variabilities are reduced?

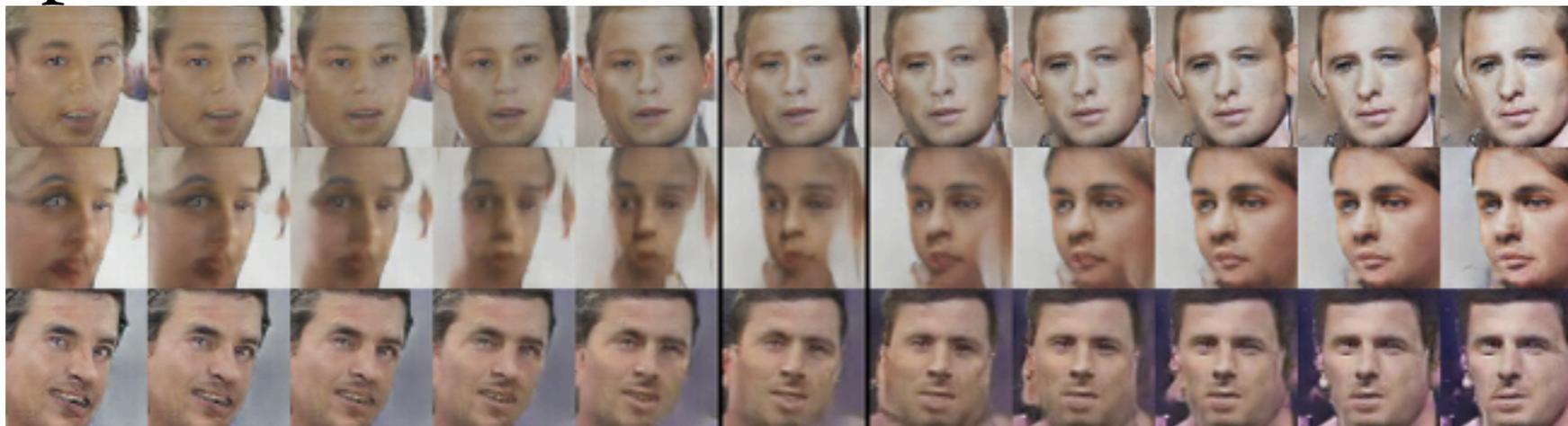
Identifying the variabilities?

- Several works showed a DeepNet exhibits some covariance:

Ref.: Understanding deep features with computer-generated imagery, M Aubry, B Russel



- Manifold of faces at a certain depth (e.g. good interpolations):



Ref.: Unsupervised Representation Learning with Deep Convolutional GAN, Radford, Metz & Chintalah

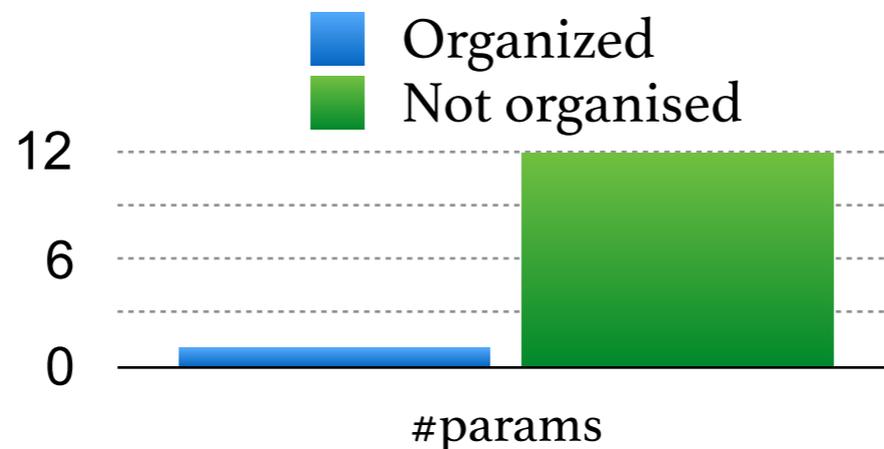
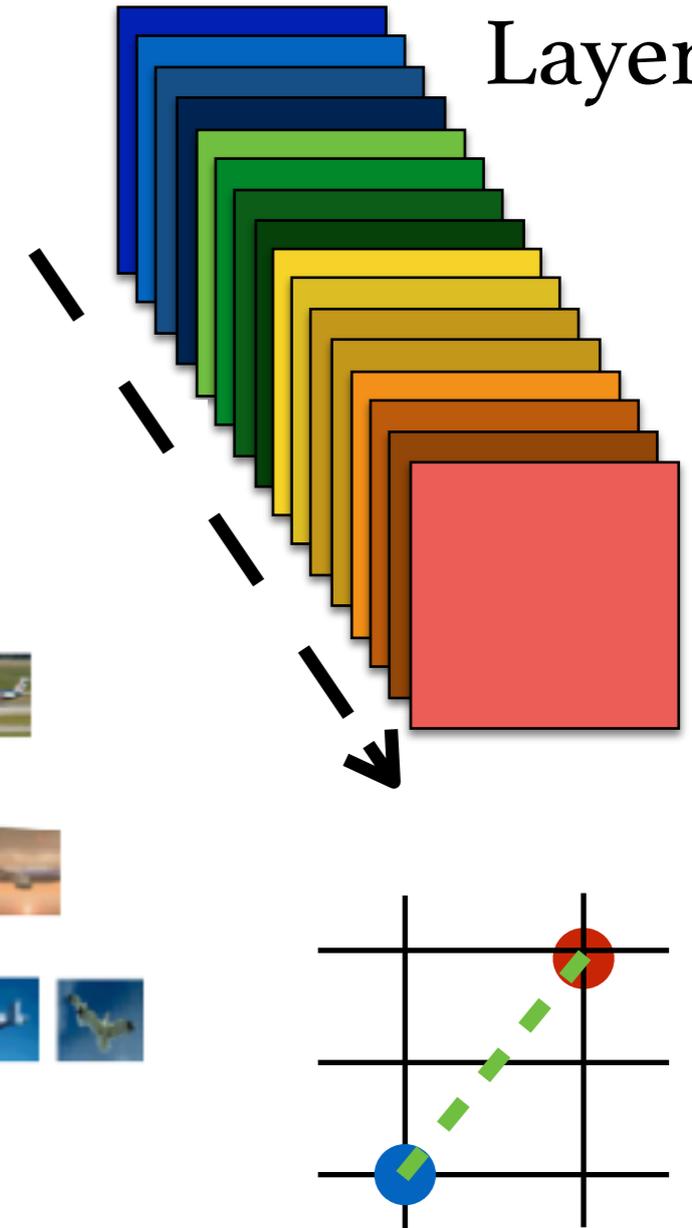
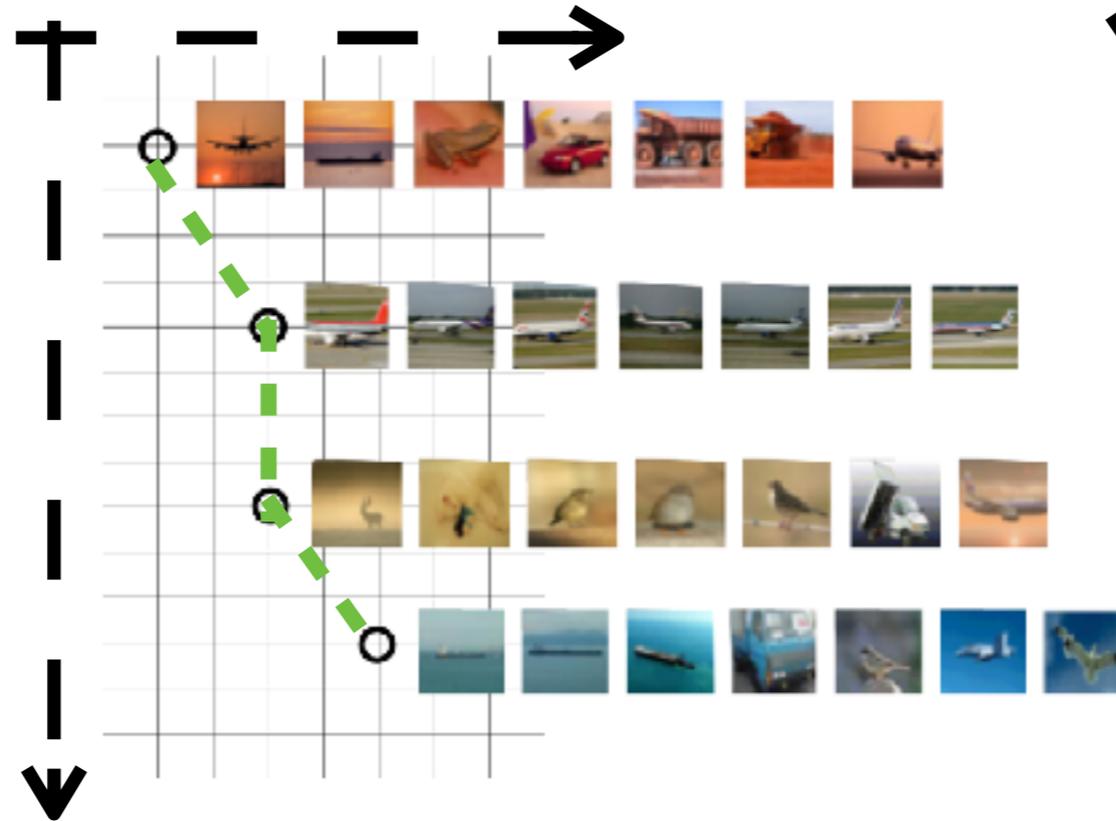
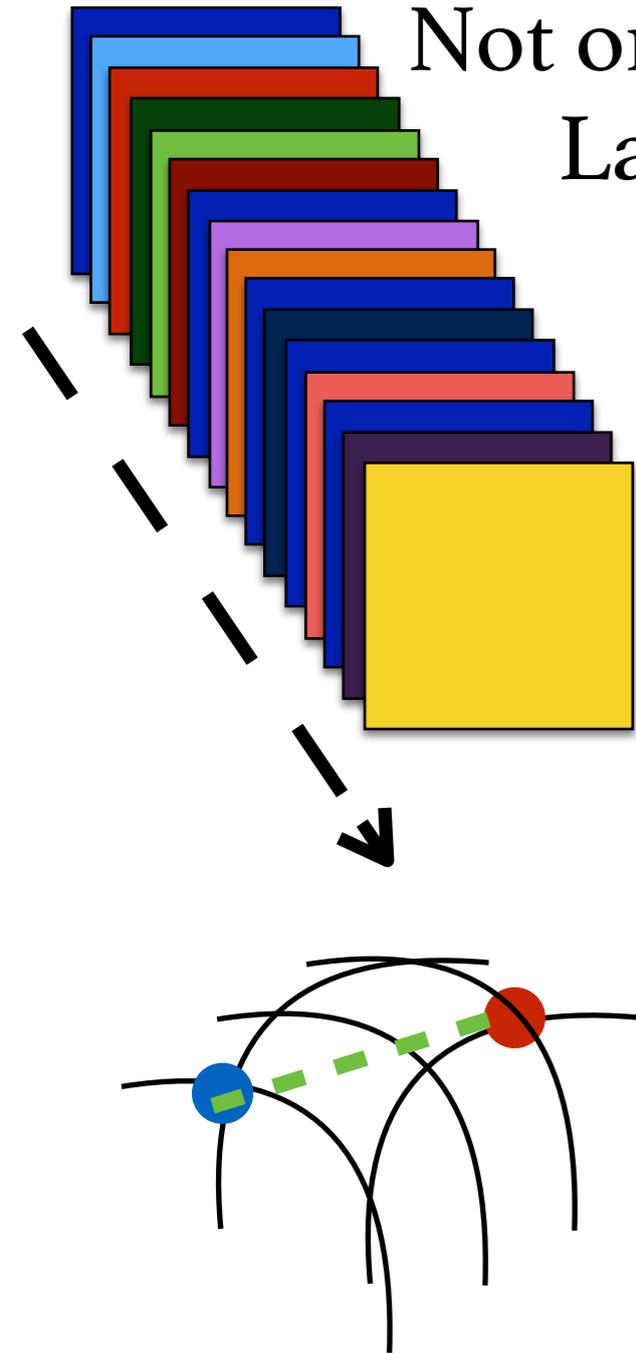
- It is hard to enumerate them...

Flattening the variability

Not organised
Layers

Organised
Layers

Defining an order
on layers of neurons



Conclusion

- Stability, generalisation results, interpretability are important aspects...
- Check the website of the team DATA:
<http://www.di.ens.fr/data/>
- Check my webpage for softwares and papers:
<http://www.di.ens.fr/~oyallon/>



Stéphane
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Jörn
Jacobsen

Thank you!