Large Scale Density-friendly Graph Decomposition via Convex Programming

Maximilien Danisch, T-H. Hubert Chan and Mauro Sozio

LTCI, Télécom ParisTech, Université Paris-Saclay, 75013, Paris, France
Department of Computer Science, The University of Hong Kong, Hong Kong, China

Motivation: mining large graphs

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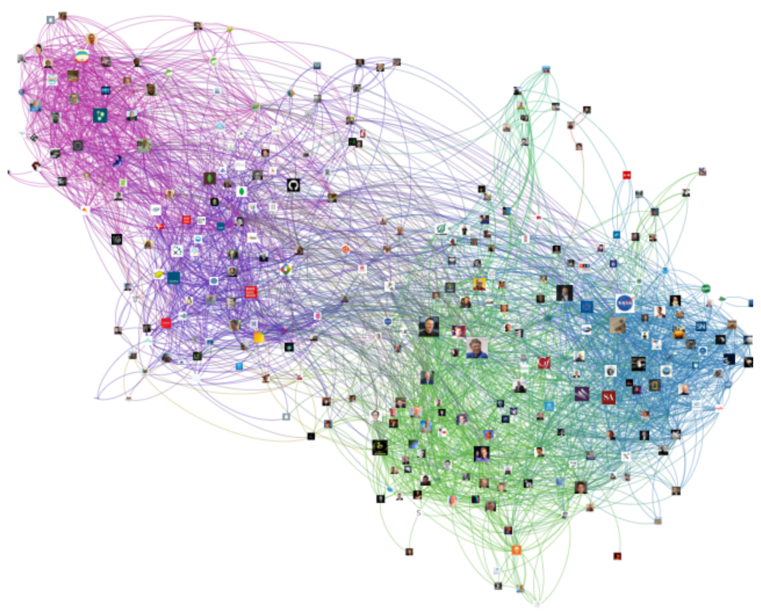

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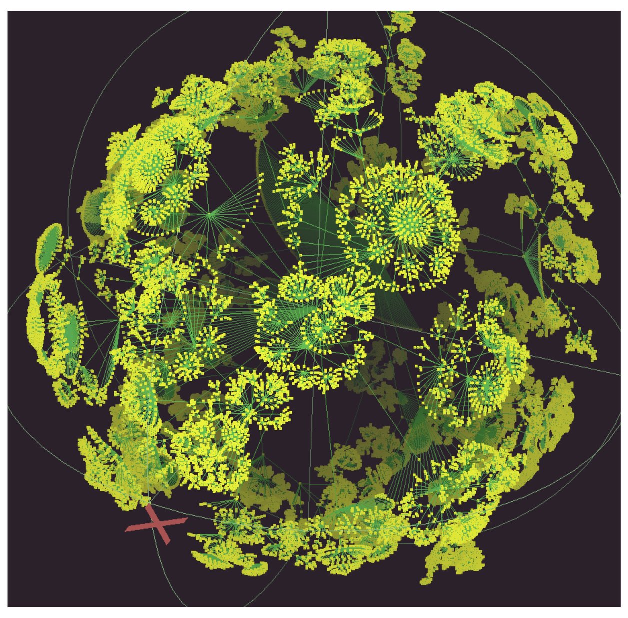


The density-friendly decomposition (Tatti and Gionis WWW2015) is interesting as it **merges two classic graph mining concepts**: **(i)** the k-core decomposition and **(ii)** the notion of dense subgraph.

Contributions

Density-friendly is now as intuitive and almost as fast to compute as PageRank.

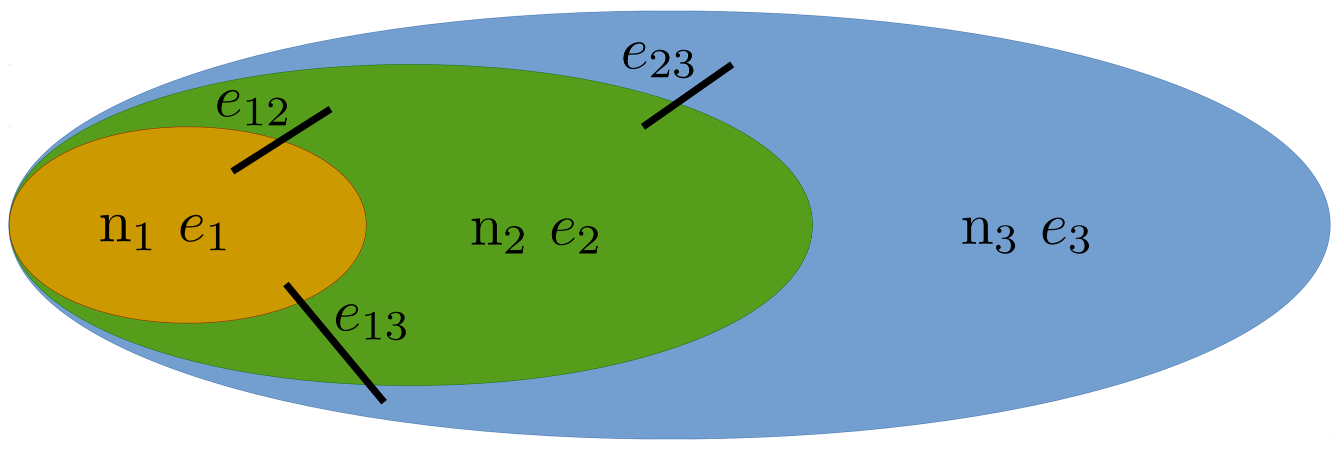
- We gave a new and **intuitive definition** of the density-friendly decomposition.
- We made an interesting link between the density-friendly decomposition and **convex programing**.
- We **scaled up** the computation of the density-friendly decomposition using the **Frank-Wolfe algorithm**.

Full paper at **WWW2017**. Code in C: `https://github.com/maxdan94`

Definition

Collection of non-overlapping sets of nodes $\{B_i\}$, such that

- B_1 maximizes $\frac{e_1}{n_1}$ and has maximum size (B_1 is the densest subgraph),
- B_2 maximizes $\frac{e_2+e_{12}}{n_2}$ and has maximum size,
- B_3 maximizes $\frac{e_3+e_{13}+e_{23}}{n_3}$ and has maximum size, ...

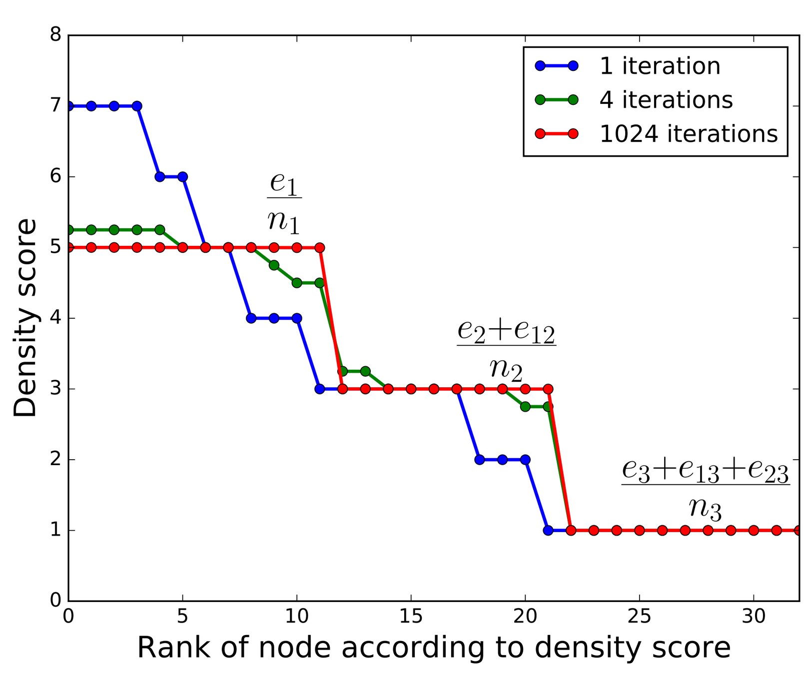


Theorem: our definition is equivalent to the one of Tatti and Gionis WWW2015.

A very simple, yet powerful, algorithm

Algorithm 1 Compute density score

```
function MKSCORE(G,t)
    Set r[i] = 0 for each node i in G
    iterate t times
        for each edge (i,j) in G do
            if r[i] ≤ r[j] then
                r[i]++
            else
                r[j]++
    return r/t
```



Theorem: rank the nodes in decreasing order of density score:

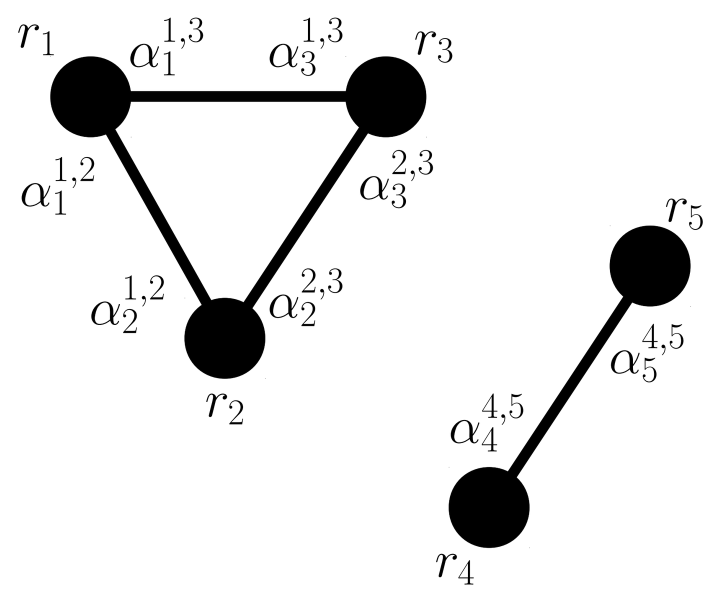
- for t large enough, the nodes in the densest subgraph are ranked first;
- for t large enough, we have a density-friendly ordering.

Theoretical analysis via convex programing

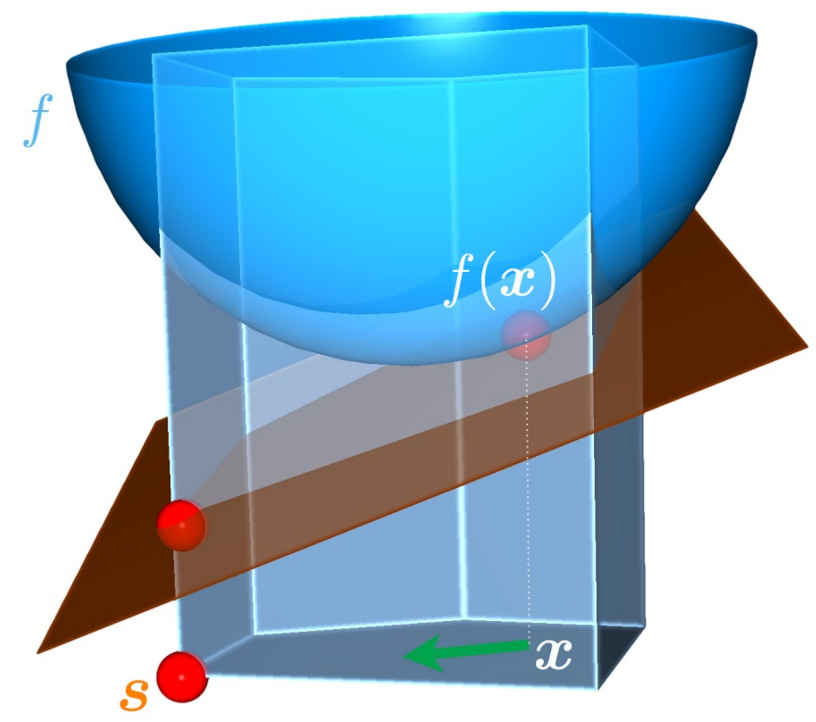
Given an edge-weighted (hyper) graph $G = (V, E, w)$, we consider the following quadratic convex programing.

$$CP(G)$$

$$\begin{aligned} \min \quad & \sum_{u \in V} r_u^2 \\ \text{s.t.} \quad & \forall u \in V,\ r_u = \sum_{e: u \in e} \alpha_u^e \\ & \forall e \in E,\ \sum_{u \in e} \alpha_u^e = w_e \\ & \forall u \in e \in E,\ \alpha_u^e \geq 0 \end{aligned}$$



The Frank-Wolfe algorithm is a **projection free gradient-descent** method which has **convergence guarantees** for convex problems.



Applying Frank-Wolfe on our quadratic convex programing leads to an algorithm very similar to our very simple algorithm.

Theorem (correctness)**:** The level sets of an optimal solution r^G to the convex programing give the density-friendly decomposition (r^G gives the densities).
Theorem (running time): After $t > \frac{4\Delta|E|}{\epsilon^2}$ iterations we have $||r^{(t)} - r^G||_2 \leq \epsilon$.

Experiments

Table 1: Our set of large graphs.

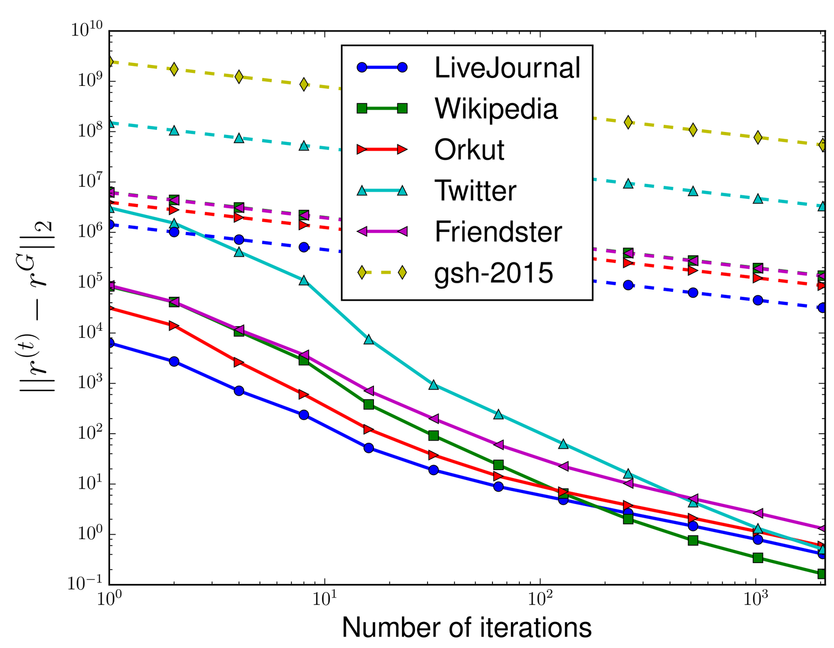
networks	n	m
LiveJournal	4,036,538	34,681,189
Wikipedia	2,080,370	42,336,692
Orkut	3,072,627	117,185,083
Twitter	52,579,683	1,614,106,500
Friendster	124,836,180	1,806,067,135
gsh-2015	988,490,691	25,690,705,119

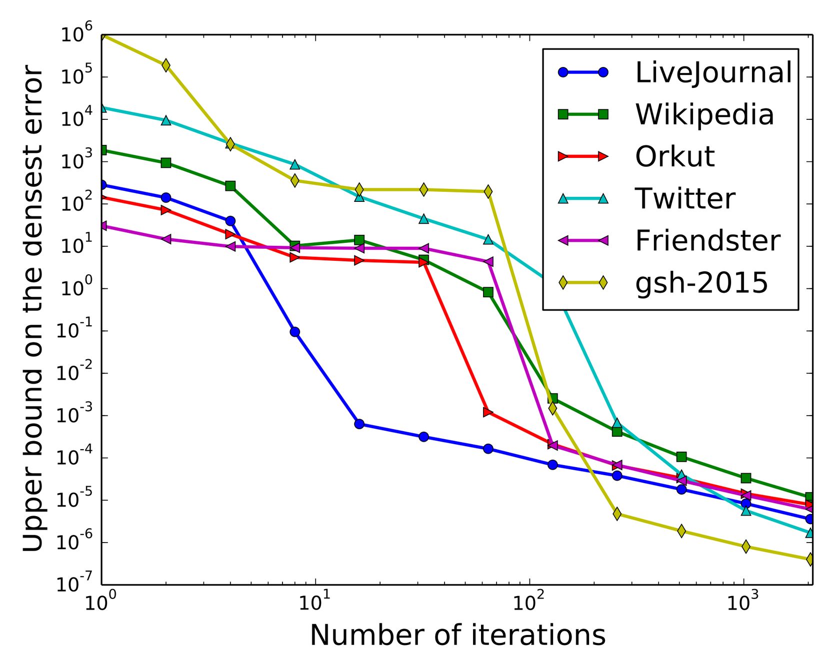
Table 2: Running time comparison of our exact algorithm to the maxflow algorithm of Tatti and Gionis WWW2015.

Networks	exact	TG15
LiveJournal	2m45s	12m02s
Wikipedia	2m14s	7m07s
Orkut	13m08s	1h02m23s
Twitter	4h57m28s	-
Friendster	5h48m27s	-

We use a machine with 64G of RAM for all networks except gsh-2015 for which we use a machine with 512G of RAM.

- Convergence in practice much faster than the worst case one
- 10^{-3} approximation of the densest subgraph within 300 iterations
- 10^{-2} approximation of the full decomposition within 1000 iterations on all networks except gsh-2015 (almost 10^{-1})
- the densest subgraph in gsh-2015 (25G edges) within 10 hours of computation





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